U.G. 2nd Semester Examination - 2022

PHYSICS

[PROGRAMME]

Course Code : PHY-G-CC-T-02

(Mathematical Physics-II)

SET–II

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Answer any **five** from the following questions: 2×5=10

a) What are the Dirichlet conditions in Fourier series?

b) Define ordinary point and singular point.

c) Write down the Laplace's equation for spherical polar coordinate.

d) Show that $erf(\infty) = 1$.

e) Distinguish between Random error and Systematic error.

[Turn Over]

f) What do you mean by even and odd function symmetry?

g) State under what conditions, a function $f(x)$ can be Fourier expanded in convergent series.

h) Define gamma function $\Gamma(n)$. Evaluate $\Gamma(-5/2)$ using $\Gamma(1/2) = \sqrt{\pi}$.

GROUP–B

2. Answer any **two** from the following questions: 5×2=10

a) Solve the following boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ given $u(0, y) = 8\, e^{-3y}$, by the method of separation of variables. 5

b) Examine the singular point for the following Bessel's equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - m^2\right)y = 0$ and also show the nature of singularity. 5

c) Expand the function $f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ A & 0 \le x < \pi \end{cases}$ in a Fourier series. 5

d) Show that $\Gamma(n+1) = n!$ and hence show $\Gamma(5/2) = (3/4)\sqrt{\pi}$. 5

GROUP–C

3. Answer any **two** from the following questions: 10×2=20

   a) State whether the following partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ is elliptic or parabolic. Find a solution to the following differential equation $\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=0$ inside an annulus bounded by the circles $x^2 + y^2 = r_1^2$ and $x^2 + y^2 = r_2^2$ that satisfies the conditions $\psi = \psi_2$ *at* $r = r_2$ and $\left(\frac{\partial \psi}{\partial r}\right)=\frac{k}{r_1}$ at $r = r_1$. 2+4

   b) Write down the expression for *erf(x)* and *erf(–x)* and show that *erf(x)* + *erf(–x)* = 0. 2+2

4. a) Show that $\beta(m, n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 4

   b) Find the Fourier series for the following function $f(x)=\begin{cases}-k, & -\pi<x<0\\ k & 0<x<\pi\end{cases}$.

   Hence show that $\frac{\pi}{4}$ = 1–1/3+1/5–1/7+1/9–... 4+2

5. a) Define Bessel's function of first kind of order n denoted by J_n and prove the recurrence formulae $xJ_n' = nJ_n - xJ_{n+1}$. 2+3

   b) Show that $g(x, t) = (1-2xt+t^2)^{-1/2}$ is the generating function of Legendre polynomials $P_n(x)$ and hence prove the recurrence relation $nP_n(x) = (2n-1)xP_{n-1}(x)-(n-1)P_{n-2}(x)$. 2+3

6. Solve any **two** from the following integral: 5×2=10

   a) $\int_0^\infty 3^{-4z^2}\,dz$

   b) $\int_0^{\pi/2}\left(\tan^3\theta+\tan^5\theta\right)e^{-\tan^2\theta}\,d\theta$

   c) $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$

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