U.G. 1st Semester Examination - 2020

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-01

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find the asymptotes of the curve $xy^2 - yx^2 = x + y + 1$.

b) If $y = \left(sin^{-1} x\right)^2$, find the value of k so that $\left(1-x^2\right)y'' - xy' + k = 0$.

c) Show that $y = x^4$ is concave upwards at the origin.

d) Find the point of inflexion of the curve $y - 3 = 6(x-2)^5$.

e) Find the envelope of the straight line $y - x = 2$.

f) Find the differential equation of all straight lines which passes through the origin.

g) Determine an integrating factor of the differential equation $\left(x^3 + y^3\right)dx - xy^2 dy = 0$.

h) If $I_n = \int_0^{\frac{\pi}{4}} tan^n\,\theta\; d\theta$, then show that $I_2 + I_0 = 1$.

i) Obtain the singular solution of the differential equation $y - px - \frac{1}{p} = 0$, where $p = \frac{dy}{dx}$.

j) Find the value of $\int_0^2\int_0^1 xy(x-y)\, dy\, dx$.

k) Find the nature of the conic $\frac{16}{r} = 4 - 5\cos\theta$.

l) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z = 71$.

m) Determine the angle of rotation of the axes so that the equation $x + y + 2 = 0$ may reduce to the form $ax + b = 0$.

2. Answer any **four** questions: 5×4=20

a) Find the envelopes of the family of ellipses $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ whose sum of the semi-axes is constant.

b) Show that all the asymptotes of the curve $r\tan 2\theta = a$ touch the circle $r=\frac{a}{2}$.

c) Find the area bounded by the curve $y^2=x^3$ and the line $y=2x$.

d) Obtain the complete primitive and singular solution of $y=px+\sqrt{1+p^2}$ where $p=\frac{dy}{dx}$.

e) Reduce the following equation to its canonical form and find the nature of the conic $3x^2+10xy+3y^2-2x-14y-5=0$.

f) Find the equation of a sphere which passes through the origin and makes equal intercepts of unit length on the axes.

3. Answer any **two** questions: 10×2=20

a) i) Show that the points of inflexion on the curve $y^2=(x-a)^2(x-b)$ lie on the line $3x+a=4b$.

ii) Solve: $\frac{d^2y}{dx^2}-y=x^2\sin x$. 5+5

b) i) Evaluate: $\underset{x\to\infty}{Lt}\left(1+\frac{1}{x^2}\right)^2$.

ii) Show that the line $x-1=y-2=z+1$, lies entirely on the surface $x^2-xy+2x+y+2z-1=0$. 5+5

c) i) The circle $x^2+y^2=a^2$ revolves round the x-axis, show that the surface area and the volume of the whole sphere generated are $4\pi a^2$ and $\frac{4}{3}\pi a^3$, respectively.

ii) Show that the plane $8x-3y-z=5$ touches the paraboloid $3x^2-2y^2=6z$. Find the point of contact. 5+5