U.G. 1st Semester Examination - 2020

PHYSICS

[HONOURS]

Course Code : PHYS-H-CC-T-1

(Mathematical Physics-I)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

   i) What do you mean by vector field?

   ii) State Gauss's Divergence theorem.

   iii) Find a unit vector that is perpendicular to $\boldsymbol{u}=(2, 3, 4)$ and $\boldsymbol{v}=(-1, 3, -5)$.

   iv) Prove that *div curl* $\boldsymbol{A}$=0.

   v) If $\operatorname{curl}\boldsymbol{A}=\frac{\partial\vec{B}}{\partial t}$, then show that div **B** is independent of t.

   vi) Find the degree and order of the differential equation $\left[1+\frac{d^2y}{dx^2}\right]^{\frac{3}{2}}=a\frac{d^2y}{dx^2}$.

   vii) What do you mean by inflection point and discontinuous function?

   viii) Give the statement of existence and uniqueness theorem for initial value problem in differential equation.

2. Answer any **two** questions: 5×2=10

   i) Solve the differential equation:

   $$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}$$ when y=0 and $\frac{dy}{dx}=1$ for x=0. Find the unit vector perpendicular to the surface $x^2+y^2-z^2=11$ at the point (4, 2,–3). 3+2

   ii) Verify stokes theorem for $\vec{F}=xz\hat{i}-y\hat{j}+x^2y\hat{k}$, where S is the surface of the region bounded by x=0, y=0 and z=0, 2x+y+2z=8, which is not included in the xz plane. 5

   iii) a) What is Dirac Delta function?

   b) Prove that $\delta\left(x^2-a^2\right)=\frac{1}{2a}\left[\delta(x+a)+\delta(x-a)\right]$ for a>0. 2+3

iv) a) If $y_1 = e^{-x} \cos x$ and $y_2 = e^{-x} \sin x$ then calculate Wronskian determinant.

b) Plot the function $f(x) = x \ln x$. 2+3

3. Answer any **two** questions: 10×2=20

i) Find the Jacobian of the transformation from rectangular Cartesian co-ordinates (x, y, z) to spherical polar co-ordinates (r, θ, φ). Also evaluate the integral

$$\iiint_V (x^2 + y^2 + z^2)\, dxdydz$$

where V is the volume of a sphere with centre at the origin and radius R. 6+4

ii) Solve the differential equations:

a) $\dfrac{dy}{dx} + x \sin 2y = x^3 \cos x^2 y$

b) $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 25y = 0$

c) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$ 3+3+4

iii) a) Show that the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ is maximum at v(–7, –7) and minimum at (3, 3).

b) The thermodynamic variables P, V, T are related $f(P, V, T)=0$ show that

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$ 5+5

iv) a) If $\vec{A}$ is irrotational, show that $\vec{A} \times \vec{r}$ is solenoidal, where $\vec{r}$ is the position vector.

b) Represent the vector $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in cylindrical coordinates (ρ, ϕ, z) and find A_ρ, A_ϕ, A_z.

c) Evaluate $\iint_S \vec{r}.\hat{n}ds$, where S is a closed surface. 2+5+3

———————