U.G. 4th Semester Examination - 2020

MATHEMATICS

[GENERIC ELECTIVE]

Course Code : MTMH-GE-T-4

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) In Z_6, which of the following equivalence classes are equal [-1], [2], [8], [5], [–2], [11], [23].

b) If a be an element of a group and $o(a) = 20$. Find the order of the element a^8.

c) If a cyclic group G has only one generator. Then prove that either $o(G) = 1$ or $o(G) = 2$.

d) Let G be a group and $a \in G$. Prove that $< a >$ is a normal subgroup of G.

e) Prove that each element in the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ is its own inverse.

f) Prove that every group of order less than 6 is abelian.

g) If G be a cyclic group of order 1000 find the number of generators of the group.

h) Prove that the center of a group G is a subgroup of the group G.

i) Prove that $(\mathbb{Q}, +)$ is non-cyclic.

j) Find all cyclic subgroups of S_3.

k) Let G be a group and H be a subgroup of G. Then prove that for any $h \in H \Rightarrow Hh = H$.

l) Show that the ring of matrices $\left\{\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in Z\right\}$ contains divisor of zero.

m) Prove that every Boolean ring is commutative.

n) Give an example of a finite ring R containing divisors of zero and a subring S of R containing no divisor of zero.

o) Let $G = \langle a \rangle$ be a group of order n. If m is a positive divisor of n, prove that $O\langle a^m \rangle = \frac{n}{m}$.

2. Answer any **four** questions: $5\times4=20$

a) Suppose that M and N are two normal subgroups of G and such that $M \cap N = \{e\}$. Show that every element of N commutes with every element M.

b) Prove that a non-commutative group of order 2n, where n is an odd prime, must have a subgroup of order n.

c) If H is a normal subgroup of G. Prove that the quotient group G/H is abelian if and only if $xyx^{-1}y^{-1} \in H$ for all $x, y \in G$.

d) Let (G, o) be a semigroup and for any two elements $a, b \in G$, each of the equations aox=b and yoa = b and has a solution in G. Then show that (G, o) is a group.

e) Prove that the characteristic of an integral domain is either zero or a prime number.

f) Prove that the set $S = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R}, b \in \mathbb{R} \right\}$ is a subring of the ring $M_2(\mathbb{R})$. Examine if the subring contains unity and divisor of zero.

3. Answer any **two** questions: 10×2=20

a) i) Let (G,o) be group and H, K are two subgroups of (G, o). Then show that $H \cup K$ is a subgroup of (G, o) if and only if either $H \subset K$ or $K \subset H$.

ii) If (G, o) be a group in which $(aob)^3 = a^3ob^3$ and $(aob)^5 = a^5ob^5$ for all $a, b \in G$, prove that the group is abelian.

b) i) If H be a subgroup of a cyclic group G, then show that the quotient group G/H is cyclic.

ii) A group G has exactly m distinct subgroups of prime order p. Prove that the total number of elements of order p in G is m(p–1).

c) i) If D is an integral domain and $a^m = b^m$, $a^n = b^n$ for all $a, b \in D$, where m, n are positive integers relatively prime, prove that a = b.

ii) If a be a fixed element in a ring R and let $C(a) = \{x \in R : xa = ax\}$. Prove that C(a) is subring of R.